

## SPECTRUM OF INTUITIONISTIC FUZZY GRAPH STRUCTURE

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**Abstract:** In this paper, the Matrix presentation in an intuitionistic fuzzy graph structure (IFGS) is discussed. The Adjacency Matrix of an intuitionistic fuzzy graph structure  $\tilde{G}$  is introduced and defined. With the help of Adjacency matrix, eigenvalue or characteristic root of an adjacency matrix  $Ad(\tilde{G})$  in IFGS are defined and studied. The concept of the spectrum of an adjacency matrix  $Ad(\tilde{G})$  in IFGS is introduced and the notion of the spectrum of IFGS is explained.

**Keywords:** Adjacency Matrix, Eigenvalue or Characteristic root, Spectrum.

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### I. INTRODUCTION:

The idea of fuzzy sets was introduced by Prof. Zadeh [11] in 1965. Rosenfeld [1] in 1975 gave the concept of fuzziness in relations and graphs. Atanassov [8] introduced the idea of intuitionistic fuzzy sets. The notion of intuitionistic fuzzy graph structure (IFGS)  $\tilde{G} = (A, B_1, B_2, \dots, B_k)$  is defined and discussed by the author in [9], [10] and [14]. The set of points when joined by arcs shows the pictorial presentation of graphs. In order to solve problems of graphs in computers, the data has to be stored in the memory of computers. This is stored with the help of Matrices. Various types of matrices are linked with graph and the spectrum of one such matrix i.e. adjacency matrix is called the spectrum of the graph [17]. The properties of the spectrum of a graph are related to the properties of the graph [17]. The area of graph theory that deals with this is called spectral graph theory [17]. The spectrum of graph was first appeared in a paper by Collatz and Sinogowitz in 1957. Presently this concept has enormous applications in various fields like

Chemistry, Computer Science, other branches of Mathematics and Physics. While solving problems in statistical physics and in combinatorial optimization problems in mathematics, Graph spectrum plays a significant role. In chemistry, in Huckel molecular orbital theory which is the theory of unsaturated conjugated molecular hydrocarbons, its numerous applications are found. In pattern recognition, in securing personal data in Databases and in modelling virus propagation in computer networks, spectrum of a graph has a major role to play. Cvetkovic and I. Gutman have discussed these applications in detail in [18]. In this paper, the Adjacency Matrix of an IFGS denoted by  $Ad(\tilde{G})$  is defined and discussed. The concept of eigenvalue or characteristic root of IFGS is also defined and then spectrum in IFGS is introduced and elucidated.

## II PRELIMINARIES

In this section, we review some definitions and results that are necessary in this paper, which are mainly taken from [3], [4], [9], [10].

**Definition (2.1) [3]:** A graph  $G$  is a pair of set  $(V, E)$ , denoted by  $G = (V, E)$ , where  $V$  is a set of vertices and  $E$  is a set of edges. Each edge in  $E$  is a pair of vertices in  $V$ . Each edge is associated with a set consisting of either one or two vertices called its endpoints.

**Definition (2.2) [3]:** An edge whose endpoints are the same is called a loop.

**Definition (2.3) [3]:** A graph without loops and parallel edges is called a simple graph.

**Definition (2.4) [3]:** Two vertices that are connected by an edge are called adjacent.

**Definition (2.5) [3]:** The adjacency matrix  $A = [a_{ij}]$  for a graph  $G = (V, E)$  is a matrix with  $n$  rows and  $n$  columns,  $n = |V|$  and its entries defined by  $a_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \in E \\ 0 & \text{otherwise} \end{cases}$ .

**Definition (2.6) [3]:** The spectrum of a matrix is defined as a set of its eigenvalues. Let  $G = (V, E)$  be a simple graph with  $n$  vertices and  $m$  edges. Let  $A = [a_{ij}]$  be the adjacency matrix of  $G$ . As in a simple graph, there can be at most 1 edge between 2 vertices. So, the entries in  $A$  are either 0 or 1. The diagonal elements are zero since there are no loops.  $A$  is symmetric and so the spectrum of  $A$  is real.

**Definition (2.7) [4] :** The eigenvalues of a matrix  $A$  are called the eigenvalues of the graph  $G$ .

**Definition (2.8) [4] :** The spectrum of  $A$  is called the spectrum of  $G$ .

**Definition (2.9) [12]:** An intuitionistic fuzzy graph is of the form  $G = (V, E)$  where

i)  $V = \{v_1, v_2, \dots, v_n\}$  such that  $\mu_1 : V \rightarrow [0,1]$  and  $\gamma_1 : V \rightarrow [0,1]$  denote the degree of membership and non membership of the element  $v_i \in V$ , respectively and  $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ , for every  $v_i \in V$ , ( $i = 1, 2, \dots, n$ ),

ii)  $E \subseteq V \times V$  where  $\mu_2 : V \times V \rightarrow [0,1]$  and  $\gamma_2 : V \times V \rightarrow [0,1]$  are such that

$$\mu_2(v_i, v_j) \leq \min\{\mu_1(v_i), \mu_1(v_j)\} \text{ and } \gamma_2(v_i, v_j) \leq \max\{\gamma_1(v_i), \gamma_1(v_j)\}$$

and  $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ , for every  $(v_i, v_j) \in E$ , ( $i, j = 1, 2, \dots, n$ ),

**Definition (2.10) [9]:** Let  $G = (V, R_1, R_2, \dots, R_k)$  be a graph structure and let  $A$  be an intuitionistic fuzzy subset (IFS) on  $V$  and  $B_1, B_2, \dots, B_k$  are intuitionistic fuzzy relations (IFR) on  $V$  which are mutually disjoint, symmetric and irreflexive such that

$$\mu_{B_i}(u, v) \leq \mu_A(u) \wedge \mu_A(v) \text{ and } \nu_{B_i}(u, v) \leq \nu_A(u) \vee \nu_A(v) \quad \forall u, v \in V \text{ and } i = 1, 2, \dots, k.$$

Then  $\tilde{G} = (A, B_1, B_2, \dots, B_k)$  is an intuitionistic fuzzy graph structure (IFGS) of  $G$ .

### III. SPECTRUM OF INTUITIONISTIC FUZZY GRAPH STRUCTURE

In this section, the Adjacency Matrix of an IFGS is defined and then eigenvalues and the spectrum are discussed.

**Definition (3.1):** A  $B_p$ -Adjacency Matrix of an intuitionistic fuzzy graph structure (IFGS) is defined as the  $B_p$ -Adjacency Matrix of the corresponding IFGS. Let  $\tilde{G} = (A, B_1, B_2, \dots, B_k)$  be an IFGS of  $G$  then  $B_p$ -Adjacency Matrix of  $\tilde{G}$  is defined as  $Ad_p(\tilde{G}) = [a_{p_{ij}}]$ ,  $a_{p_{ij}} = (\mu_{B_p}(i, j), \nu_{B_p}(i, j))$ ,  $p = 1, 2, \dots, k$  where  $\mu_{B_p}(i, j)$  represents the strength (or weightage) of relationship between  $u_i$  and  $u_j$  and  $\nu_{B_p}(i, j)$  represents the strength (or weightage) of non-relationship between  $u_i$  and  $u_j$ .

**Note (3.2):** For convenience, some notations are used here.  $\mu_{B_p}(u_i, u_j)$  is denoted with  $\mu_{B_p}(i, j)$  and  $\nu_{B_p}(u_i, u_j)$  with  $\nu_{B_p}(i, j)$ .

**Remark (3.3):** The  $B_p$ -Adjacency Matrix of an intuitionistic fuzzy graph structure can be written as two matrices one containing the matrices as membership values and the other containing the matrices as non-membership values.

$$\therefore Ad_p(\tilde{G}) = [(\mu_{B_p}(i, j), \nu_{B_p}(i, j))], \quad p = 1, 2, \dots, k$$

**Definition (3.4):** An Adjacency Matrix of an IFGS is defined as the sum of all  $B_p$ -Adjacency Matrices of the corresponding IFGS. i.e.

$$Ad(\tilde{G}) = \sum Ad_p(\tilde{G}) \quad \forall p = 1, 2, \dots, k$$

$$= Ad_1(\tilde{G}) + Ad_2(\tilde{G}) + Ad_3(\tilde{G}) + \dots + Ad_k(\tilde{G})$$

**Definition (3.5):** An IFGS which contains an edge with unordered pair of vertices is called an undirected IFGS and an IFGS which contains an edge with an ordered pair of vertices is called a directed IFGS.

**Example (3.6):** Consider an example of link structure represented by directed IFGS. The links are taken as vertices and the path between the links as edges. The strength and non-strength of the each edge are respectively represented as the membership value and the non-membership value. Consider an IFGS  $\tilde{G} = (A, B_1, B_2)$  as shown in fig 1.

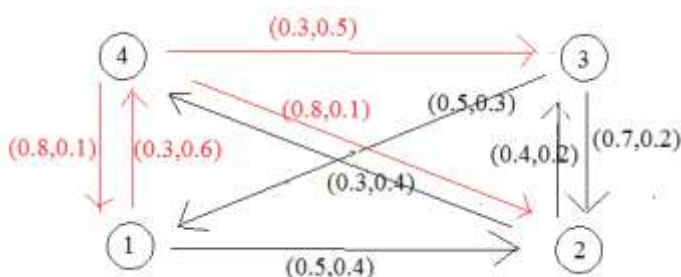


Fig 1

$$\therefore Ad_p(\tilde{G}) = [(\mu_{B_p}(i, j)), (v_{B_p}(i, j))], \quad \forall p = 1, 2, \dots, k$$

$$\therefore Ad_1(\tilde{G}) = \begin{bmatrix} 0 & a_{112} & a_{113} & a_{114} \\ a_{121} & 0 & a_{123} & a_{124} \\ a_{131} & a_{132} & 0 & a_{134} \\ a_{141} & a_{142} & a_{143} & 0 \end{bmatrix}, \text{ where } a_{p_{ij}} = (\mu_{B_p}(i, j)), (v_{B_p}(i, j)), \quad \forall p = 1, 2, \dots, k$$

$$\therefore Ad_1(\tilde{G}) = \begin{bmatrix} (0,0) & (0.5,0.4) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0.4,0.2) & (0.3,0.4) \\ (0.5,0.3) & (0.7,0.2) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0,0) \end{bmatrix} \text{ and } Ad_2(\tilde{G}) = \begin{bmatrix} (0,0) & (0,0) & (0,0) & (0.3,0.6) \\ (0,0) & (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0,0) \\ (0.8,0.1) & (0.8,0.1) & (0.3,0.5) & (0,0) \end{bmatrix}$$

$$\text{Also, } Ad(\tilde{G}) = \begin{bmatrix} (0,0) & (0.5,0.4) & (0,0) & (0.3,0.6) \\ (0,0) & (0,0) & (0.4,0.2) & (0.3,0.4) \\ (0.5,0.3) & (0.7,0.2) & (0,0) & (0,0) \\ (0.8,0.1) & (0.8,0.1) & (0.3,0.5) & (0,0) \end{bmatrix}$$

**Definition (3.7):** The Adjacency Matrix of an intuitionistic fuzzy graph structure consists of two different matrices one containing the entries as membership values and the other

containing the entries as non-membership values i.e.

$Ad(\tilde{G}) = \left( \sum \mu_{B_p}(i j), \sum \nu_{B_p}(i j) \right), \forall p$  consists of

$Ad\left[\sum \mu_{B_p}(i j)\right]$  and  $Ad\left[\sum \nu_{B_p}(i j)\right], \forall p$ .

In the above example (3.6), it can be written as

$$Ad\left[\sum \mu_{B_p}(i j)\right] = \begin{bmatrix} 0 & 0.5 & 0 & 0.3 \\ 0 & 0 & 0.4 & 0.3 \\ 0.5 & 0.7 & 0 & 0 \\ 0.8 & 0.8 & 0.3 & 0 \end{bmatrix} \quad \text{and} \quad Ad\left[\sum \nu_{B_p}(i j)\right] = \begin{bmatrix} 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0.2 & 0.4 \\ 0.3 & 0.2 & 0 & 0 \\ 0.1 & 0.1 & 0.5 & 0 \end{bmatrix}, \forall p$$

**Definition (3.8):** The eigenvalue or characteristic root of an adjacency matrix  $Ad(\tilde{G})$  is defined as  $(\theta_i, \gamma_i)$  where  $\theta_i$  is the set of eigenvalues of  $Ad\left[\sum \mu_{B_p}(i j)\right]$  and  $\gamma_i$  is the set of eigen values of  $Ad\left[\sum \nu_{B_p}(i j)\right]$ .

In the above example (3.6), eigenvalues of  $Ad\left[\sum \mu_{B_p}(i j)\right]$  are 0.670513, -0.166554, -0.25198+0.402689 i, -0.25198-0.402689 i and eigenvalues of  $Ad\left[\sum \nu_{B_p}(i j)\right]$  are 1.03868, -0.0103077, -0.514184 + 0.209431 i, -0.514184 - 0.209431 i.

**Remark (3.9):** The eigenvalue or characteristic root of an adjacency matrix  $Ad(\tilde{G})$  is called the eigenvalue or characteristic root of IFGS.

**Definition (3.10):** The spectrum of an adjacency matrix  $Ad(\tilde{G})$  is defined as  $(S_1, S_2)$ , where  $S_1$  is the set eigenvalues of  $Ad\left[\sum \mu_{B_p}(i j)\right]$  and  $S_2$  is the set of eigenvalues of  $Ad\left[\sum \nu_{B_p}(i j)\right]$ .

The spectrum of  $Ad\left[\sum \mu_{B_p}(i j)\right]$  in the above example (3.6) are given as  $S_1 = \{0.670513, -0.166554, -0.25198 + 0.402689 i, -0.25198 - 0.402689 i\}$  and the spectrum of  $Ad\left[\sum \nu_{B_p}(i j)\right]$  are given as  $S_2 = \{1.03868, -0.0103077, -0.514184 + 0.209431 i, -0.514184 - 0.209431 i\}$ .

**Remark (3.11):** The spectrum of an adjacency matrix  $Ad(\tilde{G})$  is called the spectrum of IFGS.

## V. CONCLUSION

The present paper is an important part of research work. The adjacency matrix of IFGS, eigenvalue or characteristic root and spectrum can be applied to further develop the concept of energy in terms of this matrix.

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